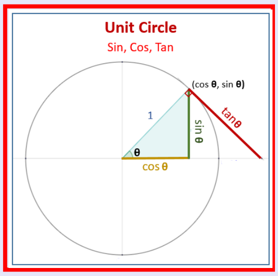
Math 241

Winter 2024

Lecture 1



Feb 19-8:47 AM

Math 241S

5 Weeks

M-Th

8:00 - 12:40

Important:

1) Arrive on time and stay for the entire time.

2) Access to Canvas. Download Canvas for student App.

3) Have a scientific calculator with you in class at all time.

4) Final Exam: See my website for details.

Review Some math:

1) Solve $3x - 2 = 7x + 8$

$3x - 7x = 8 + 2$

$-4x = 10$

$x = \frac{10}{-4}$

$\boxed{x = -\frac{5}{2}}$ $\boxed{x = -2.5}$

Solution Set $\left\{-\frac{5}{2}\right\}$

1) $\left\{-\frac{5}{2}\right\}$

2) Solve & graph. Give final solution in interval notation.

$5x - 2(x - 3) > 7x + 6$

$5x - 2x + 6 > 7x + 6$

$3x + 6 > 7x + 6$

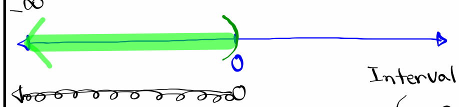
$3x - 7x > 6 - 6$

$-4x > 0$

$\frac{-4}{-4}x < \frac{0}{-4}$

$\boxed{x < 0}$

$-\infty$



$\{x \mid x < 0\}$

Interval notation

$(-\infty, 0)$

Set-Builder notation

Jan 2-8:07 AM

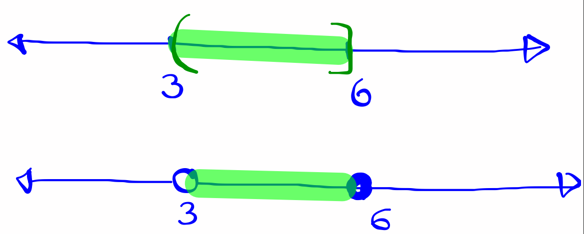
Solve & graph

$5 < 2x - 1 \leq 11$

$5 + 1 < 2x - 1 + 1 \leq 11 + 1$

$6 < 2x \leq 12$

$\boxed{3 < x \leq 6}$



Such that

S.B.N. $\{x \mid 3 < x \leq 6\}$

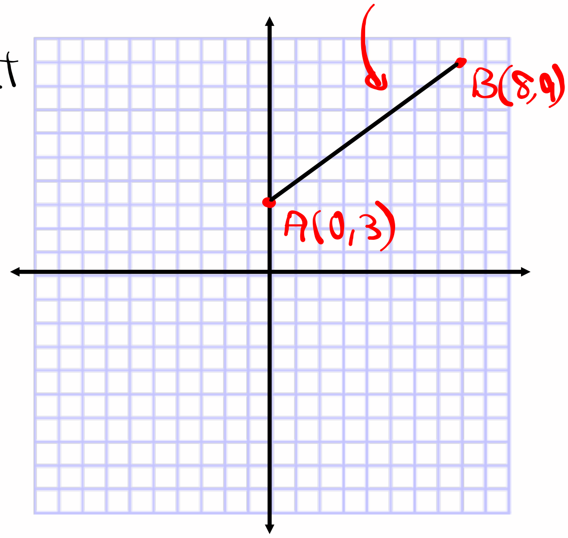
Interval notation

$(3, 6]$

Jan 2-8:19 AM

Plot A(0,3) and B(8,9)

Draw $\overline{AB}$ Line Segment



Distance from A to B.

$d(A,B)$

distance Formula

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(0-8)^2 + (3-9)^2} = \sqrt{(-8)^2 + (-6)^2}$$

$$= \sqrt{64+36} = \sqrt{100} = \boxed{10}$$

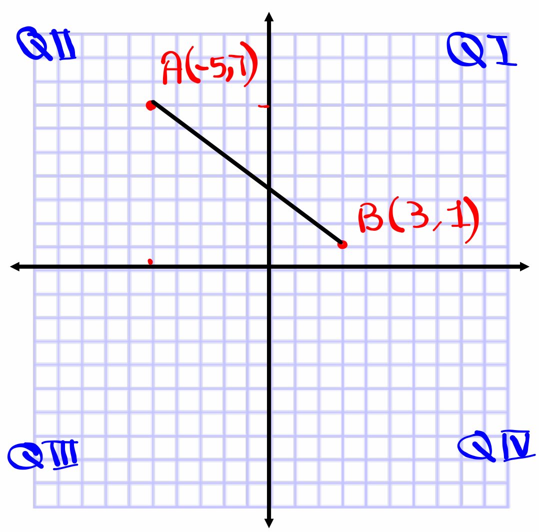
Jan 2-8:26 AM

A(−5, 7), B(3, 1)

1) Plot A & B

2) Draw $\overline{AB}$

3) $d(A,B)$



$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(-5-3)^2 + (7-1)^2}$$

$$= \sqrt{(-8)^2 + 6^2} = \sqrt{100} = \boxed{10}$$

Jan 2-8:33 AM

Use the rectangle below to find its area & its perimeter.

$A = LW$

$P = 2L + 2W$

$A = LW = (5x+4)(5x-4)$

FOIL & Simplify

$= 25x^2 - \cancel{20x} + \cancel{20x} - 16$

$= \boxed{25x^2 - 16}$

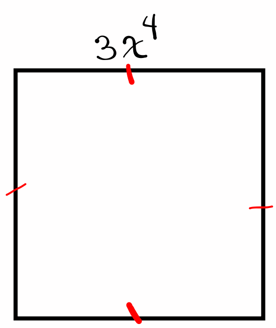
$P = 2L + 2W = 2(5x+4) + 2(5x-4)$

$= 10x + \cancel{8} + 10x - \cancel{8}$

$= \boxed{20x}$

Jan 2-8:38 AM

Use the square below to find its perimeter and area.



$P = 4S$

$A = S^2$

$P = 4S = 4(3x^4)$

$= \boxed{12x^4}$

$A = S^2 = (3x^4)^2$

$= 3^2 (x^4)^2$

$= 9x^{4 \cdot 2} = \boxed{9x^8}$

Jan 2-8:43 AM

Factor Completely: → Write in Product Form

1) $5x^2 - 10x = 5 \cdot x \cdot x - 5 \cdot 2 \cdot x$

$= \boxed{5x(x-2)}$

2) $x^2 + 6x + 8$ 1, 8; 2, 4

$= (x+2)(x+4)$

3) $2x^2 + 5x - 7 = \boxed{(2x+7)(x-1)}$

$= (2x+1)(x-7)$

$-14x + 1x = -13x$

$(2x+7)(x-1)$

$-2x + 7x = 5x$

Jan 2-8:47 AM

4) $3x^2 - 7x - 10$

-30

Product -30

Sum -7

1, -30

2, -15

$\boxed{3, -10}$

5, -6

$= 3x^2 + 3x - 10x - 10$

$= 3x(x+1) - 10(x+1)$

$= (x+1)(3x-10)$

Jan 2-8:56 AM

Special Factoring:

$A^2 + B^2$ Prime

$A^2 - B^2 = (A+B)(A-B)$

$A^3 + B^3 = (A+B)(A^2 - AB + B^2)$

$A^3 - B^3 = (A-B)(A^2 + AB + B^2)$

Jan 2-9:02 AM

Factor completely:

1) $x^2 + 25 = x^2 + 5^2 \Rightarrow \boxed{\text{Prime}}$

2) $x^2 - 36 = x^2 - 6^2 = (x+6)(x-6)$

3) $x^3 + 64 = x^3 + 4^3 = (x+4)(x^2 - 4x + 16)$

4) $x^3 - 125 = x^3 - 5^3 = (x-5)(x^2 + 5x + 25)$

Jan 2-9:05 AM

Zero - Product Rule:

If $A \cdot B = 0$, then $A = 0$ or $B = 0$

(Maybe both)

Solve

$$(2x-5)(x+8)=0$$

by Zero-Product Rule

$2x-5=0$ OR $x+8=0$

$2x=5$

$x=5/2$ $x=-8$

Solution Set $\{-8, 5/2\}$

Jan 2-9:44 AM

Solve

$$x(3x-5)=0$$

By Zero-Product Rule

$x=0$ OR $3x-5=0$

$x=0$ OR $x=5/3$

Solution Set

$\{0, 5/3\}$

Zero

Do not use

$\emptyset$ for 0.

No Solution, empty set undefined

Jan 2-9:47 AM

Solve $x^2 - 7x + 10 = 0$ by Factoring Method.

$(x-2)(x-5) = 0$ ← RHS=0

By Zero-Product Rule

$x-2=0$ OR $x-5=0$

$x=2$ $\qquad$ $x=5$ ⇒ $\{2,5\}$

Solve $5x^2 - 20 = -15x$ by Factoring Method.

$5x^2 - 20 + 15x = 0$ RHS=0

$5x^2 + 15x - 20 = 0$ order✓

$5(x^2 + 3x - 4) = 0$

$5(x+4)(x-1) = 0$

By Zero-Product Rule

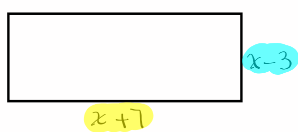
$5 \neq 0$ $\qquad$ $x+4=0$ OR $x-1=0$

$x=-4$ $\qquad$ $x=1$

Solution Set $\{-4, 1\}$

Jan 2-9:50 AM

Consider the rectangle below



Find length & width if its Area is 24 ft^2.

$A = 24$

$LW = 24$ $\qquad$ $(x+7)(x-3) = 24$

$x^2 - 3x + 7x - 21 = 24$

$x^2 + 4x - 21 - 24 = 0$

$x^2 + 4x - 45 = 0$

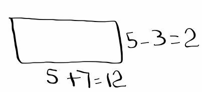
$(x+9)(x-5) = 0$

Use Zero-Product Rule

$x+9=0$ OR $x-5=0$

$\cancel{x=-9}$ $\qquad$ $x=5$

No sides can be negative.

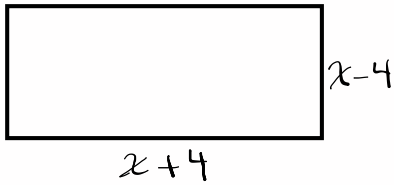


Length is 12 ft

Width is 2 ft

Jan 2-9:58 AM

Use the rectangle below



Find length and width if its area is $9cm^2$.

$(x+4)(x-4)=9$

$x^2 \quad -16=9$

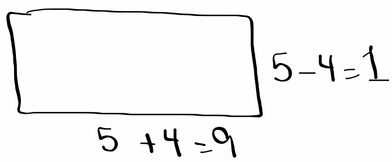
$x^2-25=0$

$(x+5)(x-5)=0$

by Zero-Product Rule

$x+5=0 \qquad x-5=0$

$\boxed{x=5}$



Length $\quad 9cm$

width $\quad 1cm$

Jan 2-10:05 AM

If $a \neq 0$, Equation $ax^2+bx+c=0$

is called Quadratic equation.

b^2-4ac is called discriminant.

The following is called Quadratic formula

$$\boxed{x=\frac{-b \pm \sqrt{b^2-4ac}}{2a}}$$

Jan 2-10:10 AM

Solve $2x^2 - 5x - 7 = 0$ by quadratic formula.

$ax^2 + bx + c = 0$

$a = 2$, $b = -5$, $c = -7$

$b^2 - 4ac = (-5)^2 - 4(2)(-7) = \boxed{81}$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-5) \pm \sqrt{81}}{2(2)}$$

$$= \frac{5 \pm 9}{4}$$

$x = \frac{5+9}{4} = \frac{14}{4} = \boxed{\frac{7}{2}}$

$x = \frac{5-9}{4} = \frac{-4}{4} = \boxed{-1}$

Soln Set $\left\{-1, \frac{7}{2}\right\}$

Jan 2-10:13 AM

Use quadratic formula to solve $x^2 + 20x + 100 = 0$. Use solution set to express final answers.

$x^2 + 20x + 100 = 0$

$a = 1$ $b = 20$ $c = 100$

$b^2 - 4ac = 20^2 - 4(1)(100) = \boxed{0}$

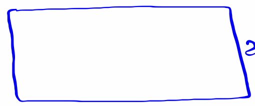
$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-20 \pm \sqrt{0}}{2(1)} = \frac{-20 \pm 0}{2} = \boxed{-10}$$

Repeated Solution

Soln Set $\{-10\}$

Jan 2-10:18 AM

find the perimeter of the rectangle below if the area is 55 in².



$$(3x+5)(2x+1)=55$$

$$6x^2+13x+5=55$$

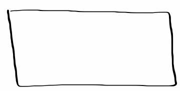
$$6x^2+13x-50=0$$

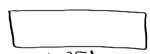
$a=6 \quad b=13 \quad c=-50$

$$b^2-4ac=13^2-4(6)(-50)=\boxed{1369}$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-13\pm\sqrt{1369}}{12}=\frac{-13\pm 37}{12}$$

$$x=\frac{-13+37}{12}=\frac{24}{12}=\boxed{2} \qquad x=\frac{-13-37}{12}=\frac{-50}{12}=\cancel{\frac{-25}{6}}$$





$$P=2L+2W=2(11)+2(5)=\boxed{32\text{ in.}}$$

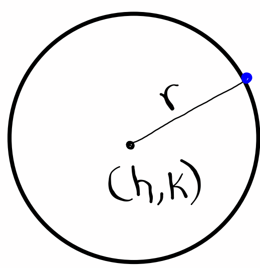
Perimeter is 32 in.

Jan 2-10:23 AM

Circle: The set of all points that are same distance from a fixed point.

Radius

Center



$$r=\sqrt{(x-h)^2+(y-k)^2}$$

If we square both sides, we get

$$\boxed{(x-h)^2+(y-k)^2=r^2}$$

Jan 2-10:34 AM

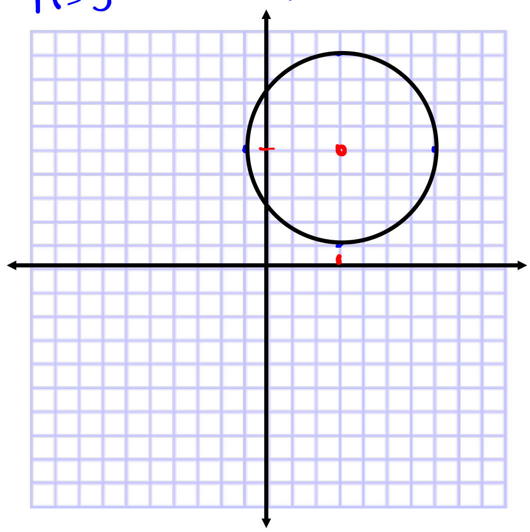
$(x-3)^2 + (y-5)^2 = 16$

$(x-h)^2 + (y-k)^2 = r^2$

$h=3$ $\quad$ $k=5$ $\quad$ $r=4$

Center $(3,5)$

radius $\quad 4$



Jan 2-10:38 AM

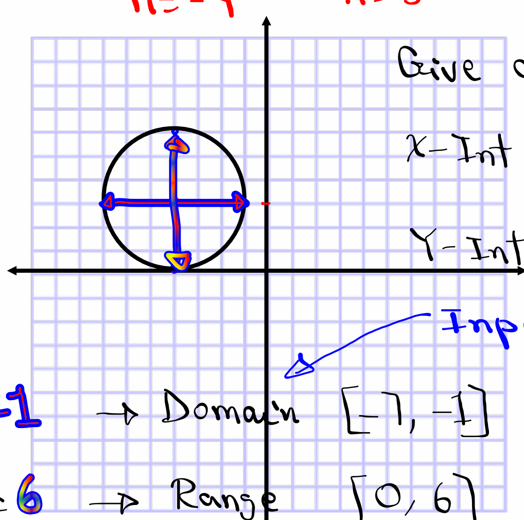
$(x+4)^2 + (y-3)^2 = 9$

Center $(-4,3)$ $\quad$ $(x-h)^2 + (y-k)^2 = r^2$

$h=-4$ $\quad$ $k=3$ $\quad$ $r=3$

Radius $\quad 3$

Draw



Give all intercepts

x-Int $\quad (-4,0)$

Y-Int $\quad$ None

Input values

$-7 \le x \le -1 \rightarrow$ Domain $[-7,-1]$

$0 \le y \le 6 \rightarrow$ Range $[0,6]$

output values

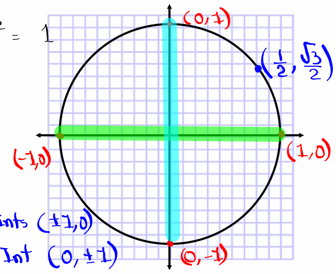
Jan 2-10:41 AM

Given $x^2 + y^2 = 1$

1) Center $(0,0)$

2) Radius 1

3) Draw



4) All intercepts x-Ints $(\pm 1, 0)$
Y-Int $(0, \pm 1)$

5) Domain & Range Domain $[-1,1]$
Range $[-1,1]$

Is $(\frac{1}{2}, \frac{1}{2})$ on the Circle? NO

$x^2 + y^2 = 1$

$(\frac{1}{2})^2 + (\frac{1}{2})^2 \stackrel{?}{=} 1 \rightarrow \frac{1}{4} + \frac{1}{4} \stackrel{?}{=} 1 \rightarrow$ NO

Is $(\frac{1}{2}, \frac{\sqrt{3}}{2})$ on the Circle? Yes

$x^2 + y^2 = 1$

$(\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2 \stackrel{?}{=} 1$

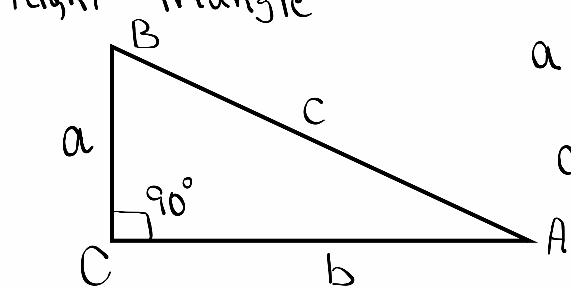
$\frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1 \checkmark$

Is $(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$ on the Circle? Yes

$(-\frac{\sqrt{2}}{2})^2 + (\frac{\sqrt{2}}{2})^2 = \frac{2}{4} + \frac{2}{4} = \frac{1}{2} + \frac{1}{2} = 1 \checkmark$

Jan 2-10:49 AM

Right Triangle

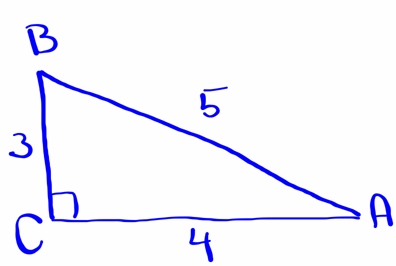


a & b are called legs.

c is called hypotenuse

$a^2 + b^2 = c^2$ is called Pythagorean thrm

a, b, and c are called Pythagorean triple.

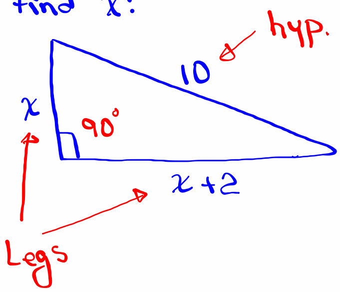


$3^2 + 4^2 = 5^2$

$9 + 16 = 25 \checkmark$

Jan 2-11:27 AM

Find x:



By Pythagorean Thrm

$$x^2 + (x+2)^2 = 10^2$$

$$x^2 + (x+2)(x+2) = 100$$

$$x^2 + x^2 + 2x + 2x + 4 = 100$$

$$2x^2 + 4x + 4 - 100 = 0$$

$$2x^2 + 4x - 96 = 0$$

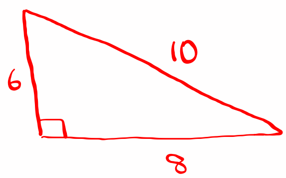
Divide by 2

$$x^2 + 2x - 48 = 0$$

$$(x+8)(x-6) = 0$$

by Z.P.R.

$\cancel{x = -8}$ $\quad x = 6$



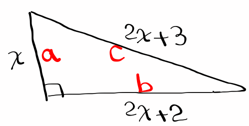
$$6^2 + 8^2 = 10^2$$

$$36 + 64 = 100$$

$$100 = 100 \checkmark$$

Jan 2-11:31 AM

Find all three sides of the shape given below



Right Triangle

$a^2 + b^2 = c^2$

$$x^2 + (2x+2)^2 = (2x+3)^2$$

$$x^2 + (2x+2)(2x+2) = (2x+3)(2x+3)$$

$$x^2 + \cancel{4x^2} + 4x + 4x + 4 = \cancel{4x^2} + 6x + 6x + 9$$

$$x^2 + 8x + 4 - 12x - 9 = 0$$

$$x^2 - 4x - 5 = 0$$

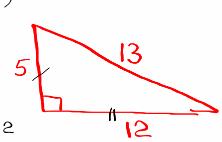
$a = 1 \quad b = -4 \quad c = -5$

$$b^2 - 4ac = (-4)^2 - 4(1)(-5) = 16 + 20 = 36$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{36}}{2} = \frac{4 \pm 6}{2}$$

$\rightarrow x = \frac{4+6}{2} = 5$

$x = \frac{4-6}{2} = \cancel{-1}$



$P = a + b + c = 5 + 12 + 13 = 30$ units

$\text{Area} = \frac{bh}{2} = \frac{5 \cdot 12}{2} = \frac{60}{2} = 30$ units^2

Jan 2-11:36 AM

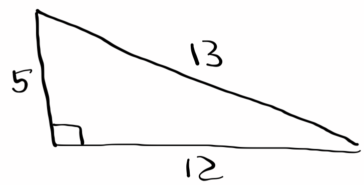
Heron's Formula

For any triangle with sides a, b, and c,

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

where

$$s = \frac{a+b+c}{2}$$



$$s = \frac{a+b+c}{2} = \frac{5+12+13}{2} = 15$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{15(15-5)(15-12)(15-13)}$$

$$= \sqrt{15 \cdot 10 \cdot 3 \cdot 2}$$

$$= \sqrt{900} = \boxed{30}$$

Jan 2-11:47 AM

Triangle ABC has 3 sides that are 4 cm, 6 cm, and 8 cm.

1) Is Triangle ABC a right Triangle?

Hint: hypotenuse is the longest side.

$$a^2 + b^2 = c^2$$

$$4^2 + 6^2 = 8^2$$

$$16 + 36 = 64$$

$$52 = 64$$

False → Not a right Triangle

2) Find its area.

Use heron's formula

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \frac{a+b+c}{2}$

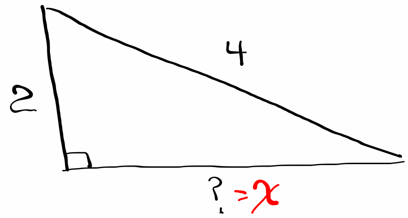
$$s = \frac{4+6+8}{2} = \frac{18}{2} = 9$$

$$\text{Area} = \sqrt{9(9-4)(9-6)(9-8)} = \sqrt{9 \cdot 5 \cdot 3 \cdot 1} = \sqrt{135}$$

$$\approx \boxed{11.6 \text{ cm}^2}$$

Jan 2-11:52 AM

Find the missing leg:



Using Pythagorean thrm

$$2^2 + x^2 = 4^2$$

$$x^2 = 12$$

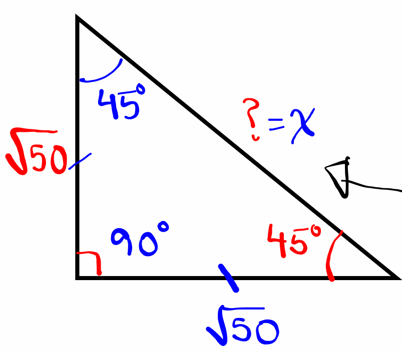
$$x = \sqrt{12}$$

$$x = \sqrt{4}\sqrt{3}$$

$$= \boxed{2\sqrt{3}}$$

Jan 2-11:58 AM

Find the hypotenuse.



Using Pythagorean thrm

$$a^2 + b^2 = c^2$$

$$(\sqrt{50})^2 + (\sqrt{50})^2 = x^2$$

$$50 + 50 = x^2$$

$$x^2 = 100$$

$$\boxed{x = 10}$$

Jan 2-12:03 PM

Some Review:

1) Solve: $2(x-3)=x+5$ → Solution Set

$2x-6=x+5$

$2x-x=5+6$ → $x=11$

1) $\{11\}$

2) Simplify: $(x-5)^2+10x$

$=(x-5)(x-5)+10x$

$=x^2-5x-5x+25+10x$

$=x^2\cancel{-10x}+25\cancel{+10x}$

$=\boxed{x^2+25}$

2) x^2+25

Jan 3-7:05 AM

3) Factor Completely:

a) $x^2+6x=x(x+6)$

a) $x(x+6)$

b) x^2+6x+9 1,9 $\boxed{3,3}$

L.C. =1

$=(x+3)(x+3)$

$=(x+3)^2$

b) $(x+3)^2$

c) x^2-x-30 1,30 2,15 3,10 $\boxed{5,6}$

$=(x+5)(x-6)$

c) $(x+5)(x-6)$

LC=4

d) $4x^2+5x-9=\underbrace{4x^2-4x}+\underbrace{9x-9}$

Product=-36 -36

Sum=5

-1, 36

-2, 18

-3, 12

$\boxed{-4,9}$

-6, 6

$=4x(x-1)+9(x-1)$

$=\boxed{(x-1)(4x+9)}$

c) $(4x+9)(x-1)$

Jan 3-7:10 AM

Use quadratic formula to solve $2x^2 - 3x - 5 = 0$

quadratic Equation $ax^2 + bx + c = 0,\ a \neq 0$

quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 2,\ b = -3,\ c = -5$

$b^2 - 4ac = (-3)^2 - 4(2)(-5)$ Discriminant

$= 9 + 40 = 49$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{-(-3) \pm \sqrt{49}}{2(2)} = \dfrac{3 \pm 7}{4}$

$x = \dfrac{3+7}{4} = \dfrac{10}{4} = \dfrac{5}{2}$

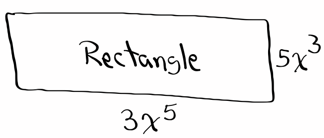
$\Rightarrow$ Solution Set $\left\{-1, \dfrac{5}{2}\right\}$

$x = \dfrac{3-7}{4} = \dfrac{-4}{4} = -1$

Jan 3-7:25 AM

Find Area & Perimeter:

1)



$A = LW$

$= 3x^5 \cdot 5x^3$

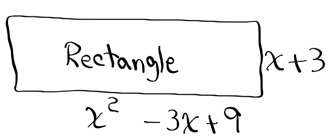
$= 15x^{5+3} = 15x^8$

$P = 2L + 2W$

$= 2(3x^5) + 2(5x^3)$

$= \boxed{6x^5 + 10x^3}$

2)



$A = LW$

$= (x+3)(x^2 - 3x + 9)$

$= x^3 - \cancel{3x^2} + \cancel{9x} + \cancel{3x^2} - \cancel{9x} + 27$

$= \boxed{x^3 + 27}$

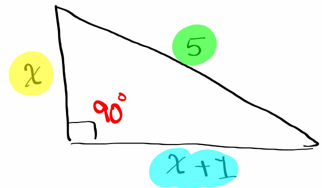
$P = 2L + 2W$

$= 2(x^2 - 3x + 9) + 2(x+3)$

$= 2x^2 - 6x + 18 + 2x + 6 = \boxed{2x^2 - 4x + 24}$

Jan 3-7:33 AM

find x:



Right Triangle

Pythagorean Thrm

$a^2 + b^2 = c^2$

$x^2 + (x+1)^2 = 5^2$

$x^2 + (x+1)(x+1) = 25$

$x^2 + x^2 + x + x + 1 - 25 = 0$

$2x^2 + 2x - 24 = 0$

Divide by 2

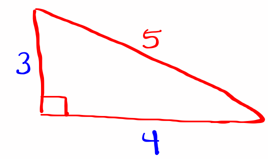
$x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$

Zero-Product Rule

$x + 4 = 0$ OR $x - 3 = 0$

$\cancel{x = -4}$ $\boxed{x = 3}$



Jan 3-7:41 AM

Solve and graph

$2x + 7 > 4x - 9$

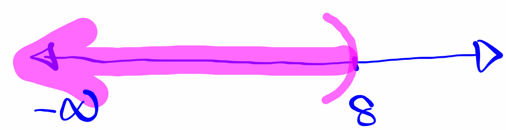
$2x - 4x > -9 - 7$

$-2x > -16$

Divide by -2

$\frac{-2}{-2}x < \frac{-16}{-2}$

$\boxed{x < 8}$



Set-Builder notation

$\{x \mid x < 8\}$

Interval notation

$(-\infty, 8)$

Jan 3-8:11 AM

Solve and graph

$$-5 < -2x + 1 \le 9$$

Hint: Isolate x in the middle.

$$-5 - 1 < -2x + 1 - 1 \le 9 - 1$$

$$-6 < -2x \le 8$$

Divide by -2

$$\frac{-6}{-2} > \frac{-2}{-2}x \ge \frac{8}{-2}$$

$$3 > x \ge -4$$

$$\boxed{-4 \le x < 3}$$

S.B.N. $\{x \mid -4 \le x < 3\}$ → Set-Builder-Notation

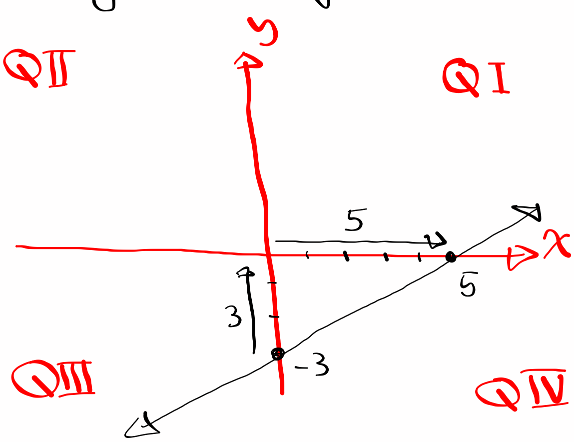
I.N. $[-4, 3)$ → Interval Notation

Jan 3-8:15 AM

Graph $3x - 5y = 15$ by completing the chart below

x	y
0	-3
5	0

Intercept Method



Increasing

$$\text{Ratio} = \frac{\text{Rise}}{\text{Run}} = \frac{3}{5} \quad \text{Slope}$$

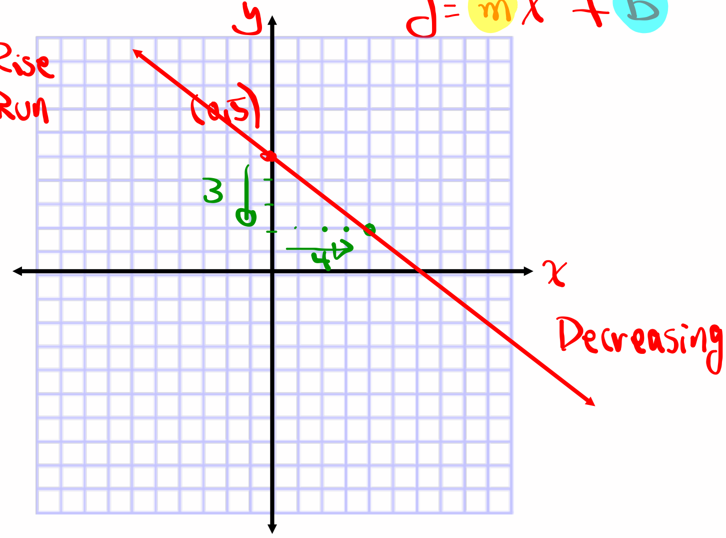
Jan 3-8:21 AM

Graph $y = \frac{-3}{4}x + 5$ using Slope-Int. Form

$y = mx + b$

Slope $m = \frac{-3}{4}$ ← Rise ← Run

Y-Int $(0,5)$



Jan 3-8:26 AM

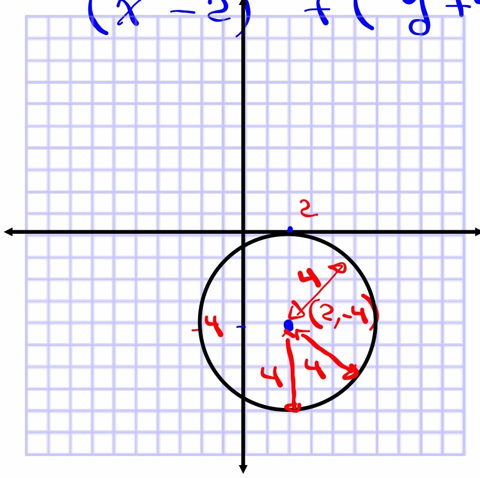
Equation of a Circle: Center (h,k)

$(x-h)^2 + (y-k)^2 = r^2$ Radius r

Ex:

$(x-2)^2 + (y+4)^2 = 16$ Center $(2,-4)$

Radius $r = 4$



Jan 3-8:30 AM

Draw $x^2 + (y-5)^2 = 25$

Center $(0,5)$

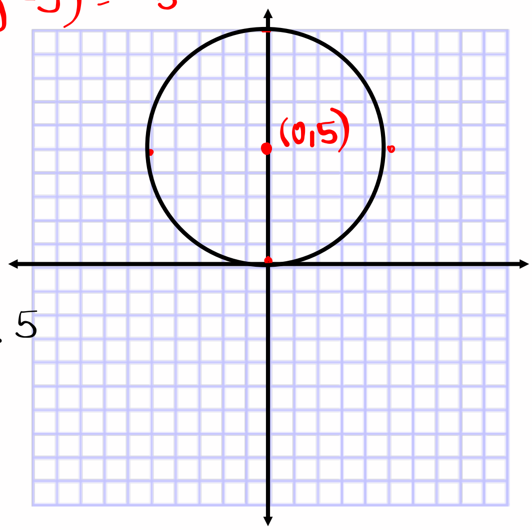
Radius 5

$$(x-0)^2 + (y-5)^2 = 5^2$$

SG 1

Due: 11:59 PM Jan. 4

Available: 11:59 PM Jan. 5



Jan 3-8:34 AM

Distance Formula between two Points:

$A(x_1, y_1)$, $B(x_2, y_2)$

$$d(A,B) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Find the distance from $\overset{A}{\boxed{(-2,3)}}$ to $\overset{B}{\boxed{(5,-4)}}$

$$d(A,B) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(-2-5)^2 + (3 - -4)^2}$$

$$= \sqrt{(-7)^2 + (7)^2} = \sqrt{49+49} = \sqrt{98} \approx 10$$

$$= \sqrt{49 \cdot 2} = \sqrt{49} \cdot \sqrt{2} = \boxed{7\sqrt{2}}$$

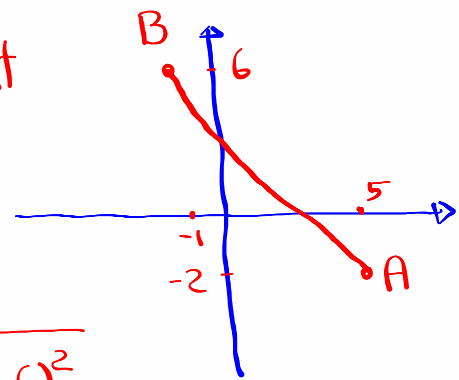
Jan 3-8:44 AM

A(5, -2) , B(-1, 6)

1) Plot A & B

2) Draw $\overline{AB}$ line Segment

3) Find d(A,B)



$d(A,B) = \sqrt{(5 - -1)^2 + (-2 - 6)^2}$

$= \sqrt{6^2 + (-8)^2} = \sqrt{36 + 64} = \sqrt{100} = \boxed{10}$

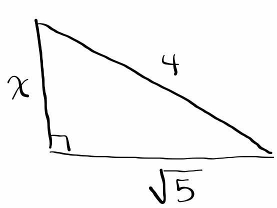
Jan 3-8:49 AM

Simplify

$\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{-1}{2}\right)^2$

$= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = \boxed{1}$

Find x:



Right Triangle

Pythagorean thrm

$x^2 + (\sqrt{5})^2 = 4^2$

$x^2 + 5 = 16$

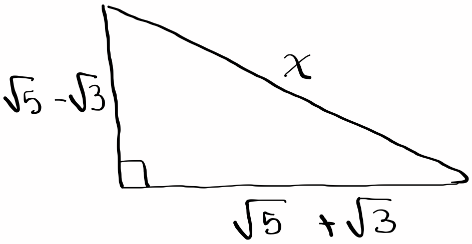
$x^2 = 11$ $\boxed{x = \sqrt{11}}$

Simplify

$\left(\frac{\sqrt{11}}{4}\right)^2 + \left(\frac{\sqrt{5}}{4}\right)^2 = \frac{11}{16} + \frac{5}{16} = \frac{16}{16} = \boxed{1}$

Jan 3-8:54 AM

Find x:



Using Pythagorean thrm

$$x^2 = (\sqrt{5}-\sqrt{3})^2 + (\sqrt{5}+\sqrt{3})^2$$

$$= (\sqrt{5}-\sqrt{3})(\sqrt{5}-\sqrt{3}) + (\sqrt{5}+\sqrt{3})(\sqrt{5}+\sqrt{3})$$

$$x^2 = \sqrt{25} - \cancel{\sqrt{15}} - \cancel{\sqrt{15}} + \sqrt{9} + \sqrt{25} + \cancel{\sqrt{15}} + \cancel{\sqrt{15}} + \sqrt{9}$$

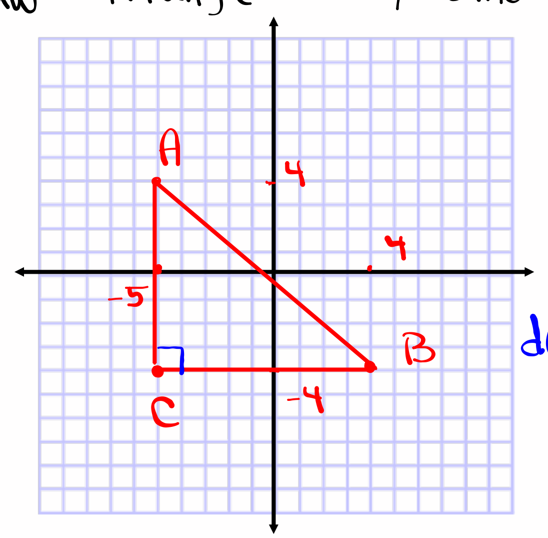
$$= 5 + 3 + 5 + 3 = 16 \qquad x^2 = 16 \qquad x = \sqrt{16}$$

$$\boxed{x = 4}$$

Jan 3-9:00 AM

$A(-5,4)$, $B(4,-4)$, $C(-5,4)$

Draw triangle ABC, find its area & perimeter



$\overline{AC} \to 8$

$\overline{BC} \to 9$

$$\text{Area} = \frac{bh}{2} = \frac{8 \cdot 9}{2} = \boxed{36}$$

$$d(A,B) = \sqrt{(-5-4)^2 + (4-(-4))^2}$$

$$= \sqrt{(-9)^2 + 8^2}$$

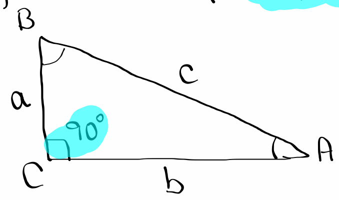
$$= \sqrt{81+64} = \sqrt{145}$$

$$P = a + b + c = 8 + 9 + \sqrt{145} = \boxed{17+\sqrt{145}} \approx 17 + 12 \approx \boxed{29}$$

Jan 3-9:05 AM

What is Trigonometry?

It is a relationship between sides and angles in any right triangle.



Using Pythagorean thrm

$a^2 + b^2 = c^2$

$\angle A + \angle B = 90^\circ$

The followings are trig. Functions:

Sine → Sin

Cosine → Cos

tangent → tan

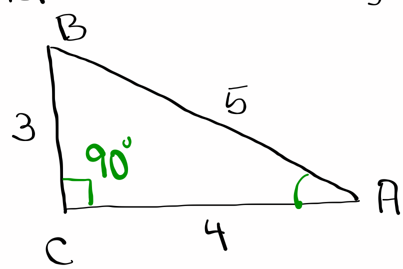
$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c}$

$\cos A = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{b}{c}$

$\tan A = \frac{\text{opposite}}{\text{Adjacent}} = \frac{a}{b}$

Jan 3-9:31 AM

Consider the triangle below



If we verify the Pythagorean Thrm $\Rightarrow \angle C = 90^\circ$

$3^2 + 4^2 = 5^2$

$9 + 16 = 25$

$25 = 25$ ✓

$\sin A = \frac{3}{5}$

$\cos A = \frac{4}{5}$

$\tan A = \frac{3}{4}$

Verify that

$\boxed{\sin^2 A + \cos^2 A = 1}$

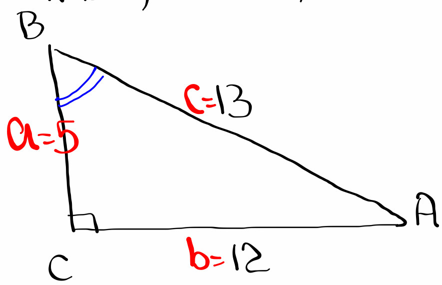
$\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2 = \frac{9}{25} + \frac{16}{25} = \frac{25}{25}$

$= 1$

Jan 3-9:37 AM

Find the missing side, then find

Sin B, Cos B, and tan B.



$a^2+b^2=c^2$

$a^2+12^2=13^2$

$a^2+144=169$

$a^2=25$

$\boxed{a=5}$

$\text{Sin B} = \frac{\text{Opp.}}{\text{Hyp.}} = \frac{12}{13}$

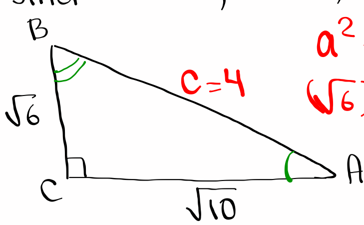
$\text{Cos B} = \frac{\text{Adj.}}{\text{Hyp.}} = \frac{5}{13}$

$\text{tan B} = \frac{\text{Opp.}}{\text{Adj.}} = \frac{12}{5}$

Avoid mixed-numbers & Decimals.

Jan 3-9:42 AM

Find Sin A, Cos B, tan A.



$a^2+b^2=c^2$

$(\sqrt{6})^2+(\sqrt{10})^2=c^2 \rightarrow c^2=16 \rightarrow \boxed{c=4}$

$\text{Sin A} = \frac{\text{Opp.}}{\text{Hyp.}} = \frac{\sqrt{6}}{4}$

$\text{tan A} = \frac{\text{Opp.}}{\text{Adj.}} = \frac{\sqrt{6}}{\sqrt{10}}$

$\text{Cos B} = \frac{\text{adj.}}{\text{Hyp.}} = \frac{\sqrt{6}}{4}$

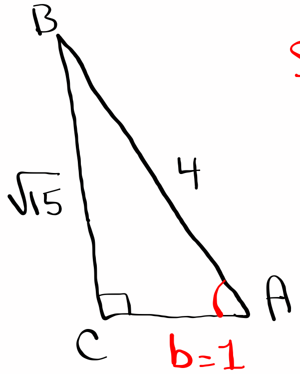
$= \frac{\cancel{\sqrt{2}}\sqrt{3}}{\cancel{\sqrt{2}}\sqrt{5}} = \frac{\sqrt{3}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{15}}{\sqrt{25}} = \boxed{\frac{\sqrt{15}}{5}}$

This answer must be rationalized.

"No radicals in the denominator"

Jan 3-9:46 AM

Find Sin A, Cos A, and tan A



$\sin A = \frac{\text{Opp.}}{\text{Hyp.}} = \frac{\sqrt{15}}{4}$

$\cos A = \frac{\text{Adj.}}{\text{Hyp.}} = \frac{1}{4}$

$a^2 + b^2 = c^2$

$(\sqrt{15})^2 + b^2 = 4^2 \rightarrow \boxed{b=1}$

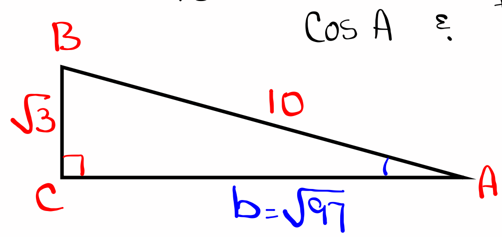
$\tan A = \frac{\text{Opp.}}{\text{adj.}} = \frac{\sqrt{15}}{1} = \boxed{\sqrt{15}}$

Jan 3-9:54 AM

Consider triangle ABC with C=90° and

$\sin A = \frac{\sqrt{3}}{10}$. Find missing Side and

Cos A & tan A.



$\sin A = \frac{\sqrt{3}}{10}$

Opp. $\sqrt{3}$

Hyp. 10

$(\sqrt{3})^2 + b^2 = 10^2$

$3 + b^2 = 100 \rightarrow b = \sqrt{97}$

$\cos A = \frac{\sqrt{97}}{10}$

$\tan A = \frac{\sqrt{3}}{\sqrt{97}} \cdot \frac{\sqrt{97}}{\sqrt{97}}$

$= \frac{\sqrt{291}}{97}$

Jan 3-9:59 AM

Now reciprocal Trig. Functions:

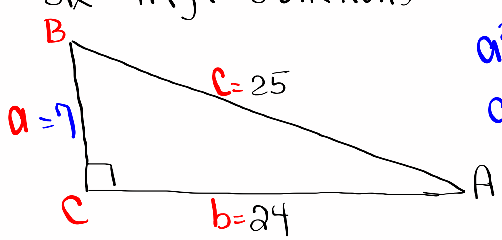
Cosecant $\to$ Csc $\to$ $\csc A = \frac{1}{\sin A}$

Secant $\to$ Sec $\to$ $\sec A = \frac{1}{\cos A}$

Cotangent $\to$ Cot $\to$ $\cot A = \frac{1}{\tan A}$

Jan 3-10:04 AM

Find the missing side, then find all six trig. functions for angle A.



$a^2 + b^2 = c^2$

$a^2 + 24^2 = 25^2$

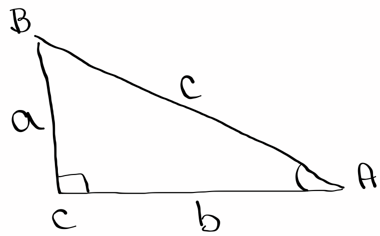
$\boxed{a = 7}$

$\sin A = \frac{7}{25}$	$\csc A = \frac{25}{7}$
$\cos A = \frac{24}{25}$	$\sec A = \frac{25}{24}$
$\tan A = \frac{7}{24}$	$\cot A = \frac{24}{7}$

Jan 3-10:07 AM

Consider the triangle below, verify that

$1 + \tan^2 A = \sec^2 A$



$\tan A = \frac{a}{b}$

$\cos A = \frac{b}{c}$, $\sec A = \frac{c}{b}$

$a^2 + b^2 = c^2$

$1 + \left(\frac{a}{b}\right)^2 = \left(\frac{c}{b}\right)^2$

$\boxed{1} + \frac{a^2}{b^2} = \frac{c^2}{b^2}$

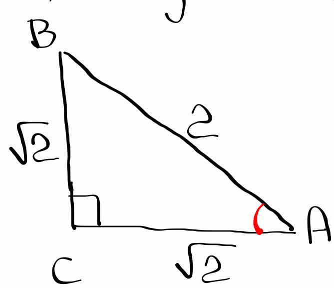
$\frac{b^2}{b^2} + \frac{a^2}{b^2} = \frac{c^2}{b^2}$

$\frac{b^2 + a^2}{b^2} = \frac{c^2}{b^2}$

$\frac{c^2}{b^2} = \frac{c^2}{b^2}$ ✓

Jan 3-10:13 AM

Use the right triangle below to find all Six trig. functions of angle A.



Let's verify the Pythagorean thrm $(\sqrt{2})^2 + (\sqrt{2})^2 = 2^2$

$2 + 2 = 4$ ✓

$\sin A = \frac{\sqrt{2}}{2}$

$\cos A = \frac{\sqrt{2}}{2}$

$\tan A = \frac{\sqrt{2}}{\sqrt{2}} = 1$

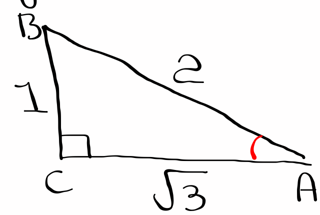
$\csc A = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{\sqrt{4}} = \sqrt{2}$

$\sec A = \frac{2}{\sqrt{2}} = \cdots = \sqrt{2}$

$\cot A = 1$

Jan 3-10:19 AM

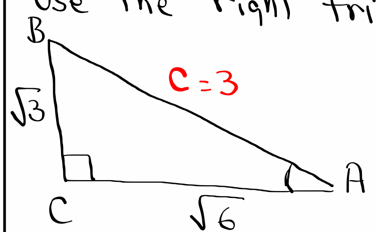
Complete the chart below using



$\sin A = \frac{1}{2}$	$\csc A = 2$
$\cos A = \frac{\sqrt{3}}{2}$	$\sec A = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$
$\tan A = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	$\cot A = \sqrt{3}$

Jan 3-10:25 AM

Use the right triangle below to complete this chart.



$\sin A = \frac{\sqrt{3}}{3}$	$\csc A = \sqrt{3}$
$\cos A = \frac{\sqrt{6}}{3}$	$\sec A = \frac{\sqrt{6}}{2}$
$\tan A = \frac{\sqrt{2}}{2}$	$\cot A = \sqrt{2}$

$(\sqrt{3})^2 + (\sqrt{6})^2 = c^2$

$3 + 6 = c^2$

$c^2 = 9$

$\boxed{c = 3}$

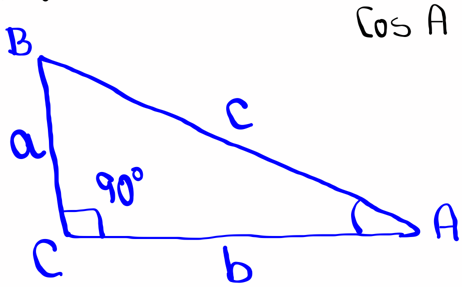
$\tan A = \frac{\sqrt{3}}{\sqrt{6}} = \frac{\cancel{\sqrt{3}} \cdot 1}{\cancel{\sqrt{3}} \cdot \sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\csc A = \frac{3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{\sqrt{9}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$

$\sec A = \frac{3}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\cancel{3}\sqrt{6}}{\cancel{6}\,2}$

Jan 3-10:30 AM

Prove $\tan A = \frac{\sin A}{\cos A}$



$\sin A = \frac{a}{c}$

$\cos A = \frac{b}{c}$

$\tan A = \frac{a}{b} = \frac{\frac{a}{c}}{\frac{b}{c}} = \frac{\sin A}{\cos A}$ ✓

Divide RHS by C, Top & bottom

Jan 3-10:39 AM

Simplify

$$(\sin A + \cos A)^2 + (\sin A - \cos A)^2$$

$$= (\sin A + \cos A)(\sin A + \cos A) +$$

$$(\sin A - \cos A)(\sin A - \cos A)$$

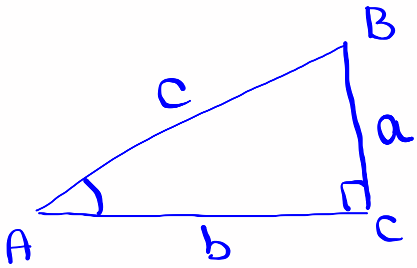
$$= \sin^2 A + \cancel{\sin A \cos A} + \cancel{\sin A \cos A} + \cos^2 A +$$

$$\sin^2 A - \cancel{\sin A \cos A} - \cancel{\sin A \cos A} + \cos^2 A$$

$$= 1 + 1 = \boxed{2}$$

Jan 3-10:43 AM

Prove $\sin^2 A + \cos^2 A = 1$



we know $a^2 + b^2 = c^2$

we also know

$\sin A = \frac{a}{c}$, $\cos A = \frac{b}{c}$

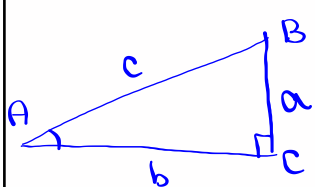
$$\sin^2 A + \cos^2 A = \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2$$

$$= \frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{a^2 + b^2}{c^2}$$

$$= \frac{c^2}{c^2} = \boxed{1}$$

Jan 3-11:02 AM

Prove $1 + \cot^2 A = \csc^2 A$



$\cot A = \frac{b}{a}$

$\tan A = \frac{a}{b}$

$\csc A = \frac{c}{a}$

$\sin A = \frac{a}{c}$

$$1 + \cot^2 A = 1 + \left(\frac{b}{a}\right)^2 = \boxed{1} + \frac{b^2}{a^2} = \boxed{\frac{a^2}{a^2}} + \frac{b^2}{a^2}$$

$1 + \cot^2 A = \csc^2 A$

$$= \frac{a^2 + b^2}{a^2}$$

$$= \frac{c^2}{a^2}$$

$$= \left(\frac{c}{a}\right)^2$$

$$= \csc^2 A \checkmark$$

Jan 3-11:06 AM

Class QZ 1:

Solve $3x^2 - 4x - 7 = 0$ by Quadratic formula.

$a=3$, $b=-4$, $c=-7$

$b^2-4ac = (-4)^2-4(3)(-7) = 16 + 84 = 100$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-4) \pm \sqrt{100}}{2(3)} = \frac{4 \pm 10}{6}$

$x = \frac{4+10}{6} = \frac{14}{6} = \boxed{\frac{7}{3}}$ $x = \frac{4-10}{6} = \frac{-6}{6} = \boxed{-1}$

$\boxed{\left\{-1, \frac{7}{3}\right\}}$

Jan 3-11:12 AM